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Maths Notes

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Discrete Mathematics

Unit 2

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Proposition Logic

Proposition Logic is defined as a Declarative Sentence that is either True or False, but not both. The Truth Value of a proposition is True (Denoted as T) if it is a True Statement, and False (denoted as F) if it's false statement.

For example:

1. The Sun Rises in the East and Sets in the West.
2. 1 + 1 = 2.
3. 'b' is a Vowel.

All the Above Sentences are propositions, where the first two are Valid (True) and the third is Invalid (False).

Some Sentences that do not have a truth value or may have more than one truth value are not propositions.

For Example:

1. What time is it?
2. Go out and Play
3. $x + 1 = 2$

The Above Sentences are not propositions as the first two do not have a truth value, and the third one may be true or false.

To Assign Value for a Statement

p, q, r

example:- p: P:

q: Q:

r: R:

we can Assign the Statement in above the variables.

★ If Statement is True $\longrightarrow$ T

If Statement is False $\longrightarrow$ F

★ Negation of a Statement (~)

If p is a proposition then Negation of p is denoted by ~p,

In another words If p is a statement therefore the negation of p is also statement but not p (~p).

i.e.

① p: ——— Statement ———

~p: ——— Statement ———
(but not p)

② let $p: 2+3 < 7$

$\sim p: 2+3 > 7$

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Truth Table for Negation

p	~p
T	F
F	T

☆ Logical Connectives & Compound Statement

[∧ Conjunction] [∨ Disjunction]

• **Conjunction**

For any Two Propositions p and q, their conjunction is denoted by $p \wedge q$, which means "p and q". The conjunction $p \wedge q$ is True when both p and q are True, otherwise False. The Truth Table of $p \wedge q$ is:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

• **Disjunction**

For any Two propositions p and q, their Disjunction is denoted by $p \vee q$. The Disjunction $p \vee q$, ~~is True when either p or q is True~~

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which means "p or q". The disjunction $p \vee q$ is True when either p or q is True, otherwise False. The Truth Table of $p \vee q$ is:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Q ⇒ Construct Truth Tables for

① $p \vee \sim q$ ② $\sim p \wedge \sim q$
③ $p \vee \sim p$ ④ $p \wedge \sim q$
⑤ $\sim(p \wedge \sim q)$ ⑥ $(p \vee q) \wedge (p \vee r)$

① Truth Table for $p \vee \sim q$

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Shot on OnePlus

② Truth Table for $\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

③ Truth Table for $p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

④ Truth Table for $p \wedge \sim q$

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

⑤ Truth Table for $\sim(p \wedge \sim q)$

p	q	~q	(p∧~q)	~(p∧~q)
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

⑥ Truth Table for ((p∨q)∧(p∨q))

p	q	(p∨q)	(p∨q)	((p∨q)∧(p∨q))
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	F

Truth Table for Three Variables

p	q	r	p∨q	p∨r	((p∨q)∧(p∨r))
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	F	F

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Conditional Statement

If p & q are two statement then Conditional Statement / Compound Statement* is " If p Then q ",
Here It is denoted by $p \Rightarrow q$.
or $p \rightarrow q$. It is also called implication.

where,
- p is Hypothesis
- q is Conclusion

If Statement p is True (T) and q is False (F) then $p \Rightarrow q$ is False Otherwise True.

Truth Table for $p \Rightarrow q$

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional Statement :
Converse, Contrapositive, and Inverse

or

Converse, Contrapositive, and Inverse of a Conditional Statement.

We can formed some new conditional statements from $p \Rightarrow q$, which have special names Converse, Contrapositive, and Inverse

(1) The Converse of $p \Rightarrow q$ is $q \Rightarrow p$.

(2) The Contrapositive of $p \Rightarrow q$ is also a conditional statement which is denoted by $\boxed{\sim q \Rightarrow \sim p}$

Or

The Contrapositive of $p \Rightarrow q$ is $\boxed{\sim q \Rightarrow \sim p}$

(3) The Inverse of $p \Rightarrow q$ is also a conditional statement which is denoted by $\boxed{\sim p \Rightarrow \sim q}$

(1) Truth Table for Converse

p	q	Converse of $p \Rightarrow q$ $= q \Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

② Truth Table for Contrapositive

p	q	$\sim p$	$\sim q$	Contrapositive of $p \Rightarrow q$ ~~=~~ $\sim q \Rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

③ Truth Table for Inverse

p	q	$\sim p$	$\sim q$	Inverse of $p \Rightarrow q$ $= \sim p \Rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

☆ Biconditional Statement (Equivalence)

If p & q are two statements the Compound Statement " p iff (if and only) is denoted by $p \Leftrightarrow$ (or) $p \leftrightarrow q$ is called Bi Conditional Statement.

If Both Statements p & q are same truth values then $p \Leftrightarrow q$ is true otherwise false.

Truth Table for Bi Conditional Statement

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

☆ Exclusive OR / ExOR / XOR

If p & q are two statements then the compound statement Exclusive-OR is denoted by $p \oplus q$.

If exactly one of the statement is true then exclusive OR of p & q is true otherwise false.

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Truth Table for $p \oplus q$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

★ **Tautology**: A Compound Statement/proposition, which is always Positive/true is called a Tautology.

e.g. $p \vee \sim p$ is a tautology

Truth Table

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

★ **Contradiction**: A Compound Proposition which is always false is called a Contradiction.

e.g. $p \wedge \sim p$ is a Contradiction

Truth Table:

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

☆ Contingency: A Compound Statement / Proposition which is neither a Tautology nor Contra-diction is called Contingency.

i.e. example :- $p \Rightarrow \sim p$ is a Contingency

Solⁿ → Truth Table

p	$\sim p$	$p \Rightarrow \sim p$
T	F	F
F	T	T

Q → Show that each of the following is a Tautology.

① $[p \land (p \Rightarrow q)] \Rightarrow q$

② $((p \Rightarrow q) \land (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$

Sol:→ ① Truth Table for ①

let $r = [p \land (p \Rightarrow q)]$

$\therefore$ find $r \Rightarrow q$

p	q	$(p \Rightarrow q)$	$p \land (p \Rightarrow q)$	$r \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$\therefore$ It is Tautology.

$[p \land (p \Rightarrow q)] \Rightarrow q$

② Truth Table for ②

let $s = ((p \Rightarrow q) \land (q \Rightarrow r))$

$t = (p \Rightarrow r)$

$\therefore$ find $s \Rightarrow t$

p	q	r	$(p \Rightarrow q)$	$(q \Rightarrow r)$	$(p \Rightarrow q) \land (q \Rightarrow r)$ ⓢ	$p \Rightarrow r$ ⓣ
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	F
F	T	F	T	F	F	F
F	F	T	T	T	T	F
F	F	F	T	T	T	F

$s \Rightarrow t$
T
T
T
T
F
T
F
F

∴ It is Contingency

$$\underbrace{((p \Rightarrow q) \land (q \Rightarrow r))}_{s} \Rightarrow \underbrace{(p \Rightarrow r)}_{t}$$

$s \Rightarrow t$

Logically Equivalence: Let r & S are two compound statements then compound statement $r \Leftrightarrow S$ is logical equivalent if $r \Leftrightarrow S$ is a Tautology.

It is Denoted by $(\equiv)$.

example:- $\boxed{r \equiv S}$

① $\sim(p \vee q) \equiv \sim p \wedge \sim q$

Sol^n:- let $\sim(p \vee q) = r$

$\sim p \wedge \sim q = S$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

$r \Leftrightarrow S$
T
T
T
T

$\therefore r \Leftrightarrow s$ is a Tautology

$\therefore r \equiv s$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

★ Predicate: It is a Sentence that Contains a finite Number of Variables.

It Becomes a Proposition when specific values are substituted for the variables.

① Example :-

- Ram is a Bachelor.
- Sohan is a Bachelor.

$p(x)$: x is a bachelor

$x \rightarrow$ Predicate Variable

$p(x) \rightarrow$ Propositional Function

② Example :- Let $p(x) : x > 3$. what are the truth values of $p(4)$ & $p(2)$.

$p(4): 4>3$, "which is true"

$p(2): 2>3$, "which is false"

③ Example :- $p(x)$: x^2 is a positive number for every real number x. Truth value ?

$p(x)$ is a true for every x.

④ Example :- $p(x)$: x^2 is a negative number for every real number x, Truth value ?

$p(x)$ is a false for every x.

☆ Quantifiers: Quantifiers are words that refer to Quantities such as some, few, many, all, none, and.

Indicate How frequently a certain statement is true.

Example :-

① Some People are rich no women is drunker

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② All Russians are Crazy.

In this examples the words Some, No, All are Quantifiers.

☆ Types of Quantifiers

① Universal Quantifier : A Universal Quantifation of predicate is the statement for all values of x, " $p(x)$ is True".

for which

The Universal Quantifier of Predicate $p(x)$ is denoted by $\forall x\, p(x)$.

where, the symbol '$\forall$' (for all) is called Universal Quantifier.

Example : The Sentence $p(x) = -(-x) = x$

$\forall x\, p(x)$.

The Quantification of $p(x)$ $\forall x p(x)$ (universal Quantification)

when x is a real number.

② **Existential Quantifier**: The Existential Quantifier of predicate $p(x)$ is the statement "There Exist values of x for which $p(x)$ is True".

The Existential Quantifier of predicate $p(x)$ is denoted by $\exists x\, p(x)$

($\exists$ There Exist)

The Symbol $\exists$ (There Exist) is called the Existential Quantifier.

Example: The Sentence $px = x+1 < 5$

"$\exists x\, p(x)$ is true."

when $p(2)$, $p(3)$

- $p(2) = 3 < 5$ "It is True"
- $p(3) = 4 < 5$ "It is True"

The Logical Equivalence below are important equivalences that should be memorized.

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① Identity Laws: $p \wedge T \equiv p$

$p \vee F \equiv p$

② Domination Laws: $p \vee T \equiv T$

$p \wedge F \equiv F$

③ Idempotent Laws: $p \vee p \equiv p$

$p \wedge p \equiv p$

④ Double Negation Law: $\sim(\sim p) \equiv p$

⑤ Commutative Law: $p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

⑥ Associative Law: $(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

⑦ Distributive Law: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

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⑧ De Morgan's Law: $\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

⑨ Absorption Law: $p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

$p \wedge (\sim p \vee q) \equiv p \wedge q$

$p \vee (\sim p \wedge q) \equiv p \vee q$

⑩ Implication Law: $(p \rightarrow q) \equiv (\sim p \vee q)$

⑪ Contrapositive Law: $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$

⑫ Tautology: $p \vee \sim p \equiv T$

⑬ Contradiction: $p \wedge \sim p \equiv F$

⑭ Equivalence: $(p \rightarrow q) \wedge (q \rightarrow p) \equiv (p \leftrightarrow q)$